BAOJ Physics

Mahdi Joharifard, et al., BAOJ Physics 2018, 3: 1
3: 028

Research

Binding Energies of Deformed Super Heavy Nuclei with Z≥105

Mahdi Joharifard[2] and Mohammad Reza Pahlavani[1*]

[*]Department of Nuclear Physics, Faculty of Sciences, University of Mazandaran

Abstract

The binding energies of super heavy nuclei around the magic nucleus Z = 114, N = 184 was studied through an improved method based on Bethe and Weizsäcker method. This modification has been done by inclusion of some terms to Bethe and Weizsäcker's initial mass formula. Since these nuclei are deformed in their ground-state, surface and coulomb terms are expanded as a function of deformation parameters. Also, shell effects are considered by including two more terms. Improved formula of the nuclear binding energy consists of seven unknown coefficients that are obtained using known experimental binding energies. The calculated binding energies for some super heavy nuclei are compared with the experimental data of AME 2012's mass table as well as theoretical results of Zhongzhou Ren and Tiekuang Dong. This comparison indicates that the calculated results using the improved method are well agreed with experimental data than the theoretical results of Zhongzhou Ren and Tiekuang Dong.

Keywords: Binding Energy; Super Heavy Nuclei; Shell Effects, Finite Range Droplet Model

PACS number: 21.10.Dr, 27.90.+b, 21.60.−n

Introduction

One of the most important properties of nuclei is its stability that is related directly to its average binding energy (binding energy per nucleon). The nuclear binding energy plays a significant role in study of nuclear mass and its other features such as nuclear stability, decay half-life and nuclear fission. Our knowledge about the decay half-life of super heavy nucleus and the height of fission barrier or its reaction cross-section is also related to the nuclear stability, directly. Therefore, study of the binding energy of nuclei was one of the important issues of nuclear physics and scientists of nuclear physics have spent a lot of time to find it out [1-12]. Be the and Weizsäcker were first scientists that performed their own studies on nuclear mass in 1930s and obtained a useful phenomenological semi-empirical relationship for nuclear mass as a sum of its various possible energies [1-2]. Following their studies, Bohr and Wheeler interpreted the energy released from neutron induced fission of ^{235}U (^{236}U compound isotope) using this approach [13]. At present one of the nuclear physics necessity is to find out the mass and binding energies of super heavy nuclei that are produced in heavy ion fusion-fission reactions [4-20]. Various methods have been introduced in order to obtain the binding energies of the nucleus in different region of mass numbers.

Others attempt to study the ground-state properties of nuclei are performed using the finite range droplet model (FRDM), folded Yukawa single particle potential and self-consistent mean field [8-11]. Recently, AdS/CFT correspondence holography model is used to calculate binding energies of light nuclei [20-23]. Considering 1 MeV uncertainty for theoretical calculations of binding energy of super heavy nuclei [24-25], is seems necessary to obtain a more accurate equation to calculate binding energy. In the shell-model representation, those nuclei with closed shell Z or N are called "magic" and also when both Z and N numbers are magic, the nucleus is called "double magic". So biczewski et al. [26] improved original mass formula to consider closed shell properties of ^{270}Hs, ^{208}Pand ^{298}114 heavy and super heavy isotopes. In addition, Nilsson et al.[27] obtained2×10^{19} year for the half-life of spontaneous fission of^{298}114superheavy nucleus, which is much higher than the fission half-life of its neighbor isotopes. This means that the fission barrier for this nuclei is significantly different compared to its surrounded nuclei [28]. Therefore it seems necessary to consider this feature as shell effects for super heavy nuclei around ^{298}114. This study attempts to find an improved relationship for binding energies of super heavy nuclei withZ≥105. This paper is organized as follows. In section 2 the improved version of binding energy formalism is presented with seven adjustable parameters. The calculated binding energies for 59 super heavy nuclei calculated using this improved approach are presented and compared with theoretical

***Corresponding Author**: Mohammad Reza Pahlavani, Department of Nuclear Physics, Faculty of Sciences, University of Mazandaran, P.O.Box 47415-416, Babolsar, Iran Tel: +981135302498; E-mail: m.pahlavani@umz.ac.ir

Sub Date: January 20th 2018, **Acc Date:** February 6th 2018 , **Pub Date**: February 8th 2018.

Citation: Mahdi Joharifard and Mohammad Reza Pahlavani (2018) Binding Energies of Deformed Super Heavy Nuclei with Z≥105. BAOJ Physics 3: 28.

results of Zhongzhou Ren and TiekuangDong as well as the experimental data of AME 2012 mass table in section 3. Finally, concluding remarks are given in section 4.

Definition of Improved Approach

As it is mentioned in previous section, first phenomenological formula that presented for calculating of binding energy in the base of similarity between the liquid drop and nuclear material was proposed by Bethe and Weizsäcker[1-3]:

$$B(A,Z)=a_vA-a_sA^{\frac{2}{3}}-a_cZ^2A^{-\frac{1}{3}}-a_a\left(\frac{A}{2}-Z\right)^2A^{-1}+a_p\delta A^{-\frac{1}{2}}. \qquad (1)$$

Where a_v, a_s, a_c, a_a and are volume, surface, coulomb, asymmetry and pairing coefficients, respectively. These parameters can be adjusted using known binding energies of at least five isotopes. δ =1, 0 and -1 are simply agreed for even-even, even-odd and/or odd-even and odd-odd, Z - N nucleus, respectively. Amounts of these coefficients are related to selection of known experimental data of binding energies. Therefore there are many selections of these coefficients. In a study by Zhongzhou Ren and TiekuangDong [29] on heavy nuclei with Z ≥ 90, the following coefficients were obtained without considering the shell effects and deformation

$$\begin{cases} a_v = 15.7226MeV \\ a_s = 17.7523MeV \\ a_c = 0.7062MeV \\ a_a = 96.2350MeV \\ a_p = 10.6028MeV \end{cases}$$

Eq. (1) is improved in advance to consider deformation and shell effects for super heavy nuclei. Because of conservation of volume, the volume of nuclei is not change by deformation. Therefore the portion of volume in binding energy is

$$B_V = a_vA.$$

Considering of deformation on surface energy up to second order convert it to [31,32]

$$B_s = a_sA^{\frac{2}{3}}\left(1+\frac{2}{5}a_2{}^2-\frac{4}{105}a_2{}^3+\ldots\right).$$

The effects of deformation also have an impact on Coulomb energy as well. Thus, by considering deformation, Coulomb energy is rewritten as [33],

$$B_c = a_cZ^2A^{-\frac{1}{3}}\left(1-\frac{1}{5}a_2{}^2-\frac{4}{105}a_2{}^3+\ldots\right).$$

The asymmetry energy not affected by deformation, so we have,

$$B_a = a_a\left(\frac{A}{2}-Z\right)^2A^{-1}.$$

Also, the pairing energy not changed considerably by deformation,

$$B_p = a_p\delta A^{-\frac{1}{2}}.$$

Considerable difference between theoretical and experimental data for super heavy nuclei around closed shell Z=114 and N=184 indicate that the theoretical formula of binding energy should be revise [30]. Studies on fission barrier [34] indicate that the fission barriers high for nuclei around ^{298}114 are growth considerably than other super heavy nuclei in this region.

Figure 1 clearly verifies this criterion. Therefore we revised the theoretical formula of binding energy by including two terms to emphasis on the shell effects as:

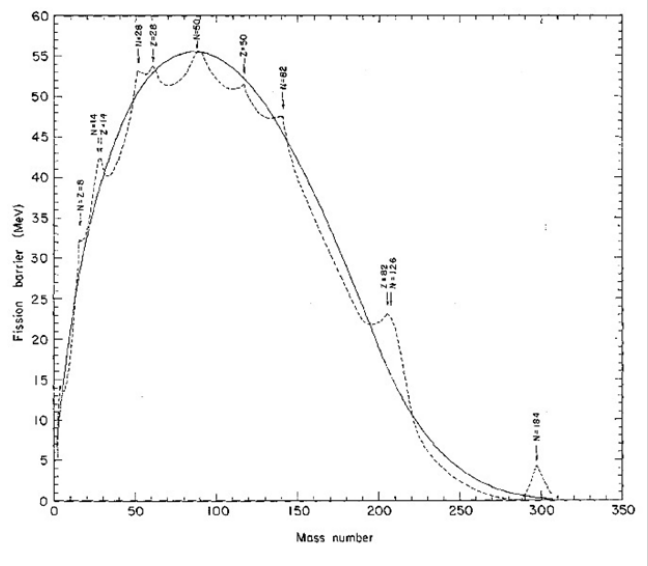

Figure 1. The fission barrier energy, in MeV as a function of mass number (A), for nuclei in the valley of stability. The smooth curve is the results of liquid-drop model. The irregular dashed curve is calculated from Meyers and Swiatecki mass formula that shows shell effects clearly. Nuclei with N ~ 50 (A ~ 90) should require the greatest amount of energy for their disintegration, lighter and heavier elements being more easily disrupted into comparable fragments [28].

$$B_{Shell} = a_6\left|1-\frac{298}{A}\right|-a_7\left|1-\frac{184}{N}\right|.$$

By including these new terms and effects of deformation, the improved version of Bethe and Weizsäcker original equation for calculating binding energy of super heavy nuclei around Z=114 and N=184 is rewritten as

$$\begin{aligned} B(A,Z) &= a_vA-a_sA^{\frac{2}{3}}\left(1+\frac{2}{5}a_2{}^2-\frac{4}{105}a_2{}^3+\ldots\right) \\ &-a_cZ^2A^{-\frac{1}{3}}\left(1-\frac{1}{5}a_2{}^2-\frac{4}{105}a_2{}^3+\ldots\right) \\ &-a_a\left(\frac{A}{2}-Z\right)^2A^{-1}-a_p\delta A^{-\frac{1}{2}} \\ &+a_6\left|1-\frac{298}{A}\right|-a_7\left|1-\frac{184}{N}\right|. \end{aligned}$$

Following coefficients are obtained for this formula through fitting method with experimental data,

$$\begin{cases} a_v = 15.6446 MeV \\ a_s = 16.9970 MeV \\ a_c = 0.71197 MeV \\ a_a = 96.6732 MeV \\ a_6 = 55.26028 MeV \\ a_7 = 57.9814 MeV. \end{cases}$$

And $a_p = 3MeV$ is agreed for pairing energy coefficient. Also, the following values were used for δ:

$$ä = \begin{cases} 4.22 Z-even, N-even \\ 1 Z-even, N-odd \\ 0 Z-odd, N-even \\ -2.66 Z-odd, N-odd \end{cases}$$

This improved formula has been used to calculate the binding energies of 59 super heavy nuclei.

Numerical Results and Discussion

The calculated results for binding energies of 59 super heavy nuclei along with the results of Zhongzhou Ren and Tiekuang Dong and the experimental data [12]. Are presented in Table 1. This table shows that the calculated binding energies through this approach have been improved over the calculated results of Zhongzhou Ren and Tiekuang Dong and the obtained results are agreed well with the experimental data.

Average deviation and root mean square deviation of binding energies were calculated to indicate agreements between our results and experimental data than other theoretical results as,

$$\langle \sigma \rangle = \frac{\sum_{i=1}^{59} (B_{\exp i} - B_{Cali})}{59} = 0.524 MeV$$

$$\sqrt{\sigma^2} = \left(\frac{\sum_{i=1}^{59} \left(B_{\exp i} - B_{Cali} \right)^2}{59} \right)^{\frac{1}{2}} = 0.597 MeV$$

Figure 2 : shows the calculated results for 59 nuclei along with experimental results [12]. In figure 3 the average binding energies for 59 nuclei are compared with experimental data [12]. The deviations between experimental data and calculated binding energies for 59nuclei are presented in Figure 4. As it can be seen from these figures, the results of this approach are compatible with experimental data than theoretical results of Zhongzhou Ren and Tiekuang Dong.

Conclusion

Considering the fact that there is limited experimental information regarding super heavy nuclei, our main objective was to introduce an improved version to calculate accurate binding energies for super heavy nuclei. Two new terms originating from shell effect were included to Bethe and Weizsäcker's original equation. Moreover, surface and coulomb energies of liquid drop formula are deformed as a function of deformation parameter up to second order. Seven unknown coefficients of the improved equation were obtained through fitting with experimental data. This improved approach is used to calculate the binding energies of 59 super heavy nuclei. Obtained results were compared with theoretical results of Zhongzhou Ren and Tiekuang Dong as well as experimental data. The average deviation between theoretical results and experimental data is obtained equal to 0.524 MeV that is illustrating the acceptable accuracy of new improved approach.

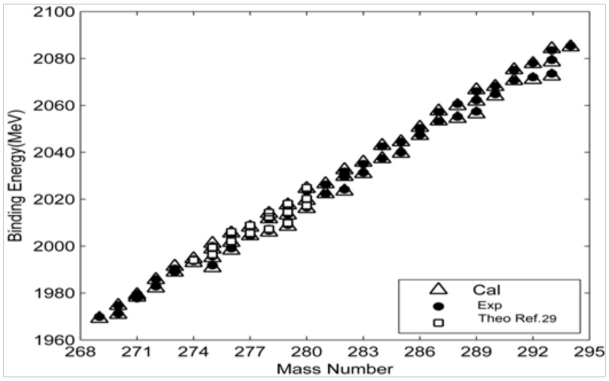

Figure 2. Binding energies of 59 nuclei compared with experimental data and results of other theoretical methods.

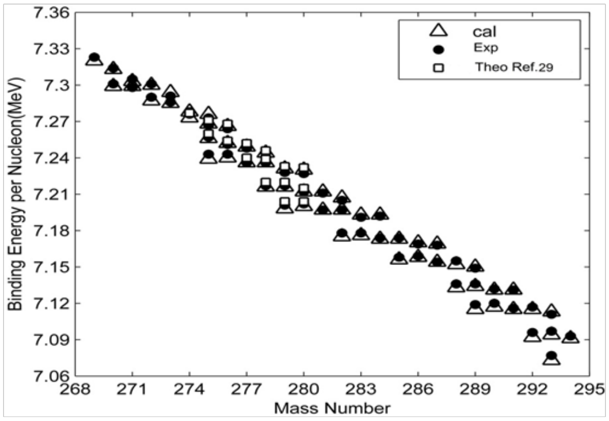

Figure 3. Average binding energies of 59 nuclei compared with results of other theoretical methods and experimental data.

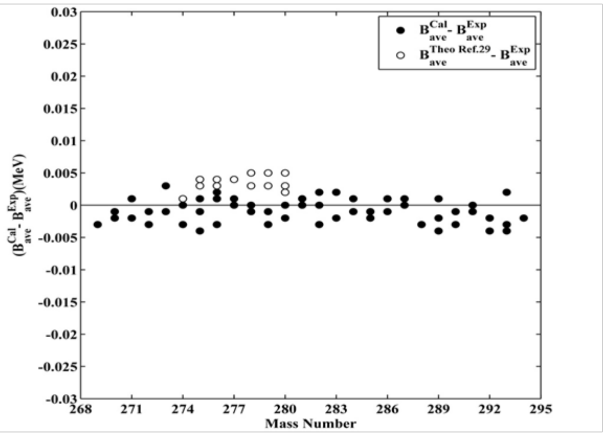

Figure 4. The deviations between experimental and calculated binding energies as a function of mass number.

Table 1. Calculated binding energies for 59 super heavy nuclei along with theoretical and experimental data.

Z	N	A	Elt.	$B^{CAL.}_{(MeV)}$	$B^{EXP.}_{(MeV)}$	$B^{CAL.REF.[12]}_{(MeV)}$	$B^{CAL.}_{ave(keV)}$	$B^{EXP.}_{ave.(keV)}$	$B^{CAL.\ REF.[12]}_{ave.(keV)}$
105	164	269	Db	1969.106067	1969.887		7320	7323	
105	165	270	Db	1974.623	1974.78		7313	7314	
106	165	271	Sg	1979.243519	1979.655		7303	7305	
106	166	272	Sg	1985.670635	1985.872		7300	7301	
106	167	273	Sg	1991.320192	1990.443		7294	7291	
107	163	270	Bh	1970.846677	1971.28		7299	7301	
107	164	271	Bh	1978.177785	1977.758		7299	7298	
107	165	272	Bh	1982.084731	1982.88		7287	7290	
107	166	273	Bh	1988.859482	1989.078		7285	7286	
107	167	274	Bh	1994.551196	1994.174		7278	7278	
107	168	275	Bh	2001.091304	2000.075		7276	7273	
108	166	274	Hs	1992.89976	1993.624	1994.014	7273	7276	7277
108	167	275	Hs	1998.682605	1998.425	1999.468	7268	7267	7271
108	168	276	Hs	2005.513782	2004.864	2006.069	7266	7264	7268
109	166	275	Mt	1994.987672	1995.675	1996.430	7256	7257	7260
109	167	276	Mt	2001.518211	2001.276	2002.078	7252	7251	7254
109	168	277	Mt	2007.988945	2007.696	2008.870	7249	7248	7252
109	169	278	Mt	2013.8358	2012.998	2014.294	7244	7241	7246
110	165	275	Ds	1990.730051	1991.825		7239	7243	
110	166	276	Ds	1998.182942	1999.068		7240	7243	
110	167	277	Ds	2004.353034	2004.372	2005.523	7236	7236	7240
110	168	278	Ds	2011.565406	2011.608	2012.504	7236	7236	7239
110	169	279	Ds	2017.501792	2016.612	2018.118	7231	7228	7233
110	170	280	Ds	2024.478874	2023.56	2024.870	7230	7227	7232
111	167	278	Rg	2005.903234	2006.604	2007.291	7216	7217	7220
111	168	279	Rg	2013.206421	2013.543	2014.463	7216	7217	7220
111	169	280	Rg	2019.435951	2019.36	2020.268	7212	7212	7215
111	170	281	Rg	2026.502409	2026.291		7212	7211	
111	171	282	Rg	2032.501171	2031.81		7207	7205	
112	167	279	Cn	2008.357044	2009.079	2009.896	7198	7201	7204
112	168	280	Cn	2015.949574	2016.56	2017.255	7200	7202	7204
112	169	281	Cn	2022.2684	2022.357		7197	7197	
112	170	282	Cn	2029.622158	2029.554		7197	7197	
112	171	283	Cn	2035.708877	2035.053		7193	7191	
112	172	284	Cn	2042.828835	2042.528		7193	7192	
113	169	282	Ed	2023.380036	2024.196		7175	7178	
113	170	283	Ed	2030.822932	2031.374		7176	7178	
113	171	284	Ed	2037.199747	2037.416		7173	7174	
113	172	285	Ed	2044.4075	2044.59		7173	7174	
113	173	286	Ed	2050.555048	2050.334		7170	7169	
113	174	287	Ed	2057.532682	2057.216		7169	7168	
114	171	285	Fl	2039.5004	2040.03		7156	7158	
114	172	286	Fl	2047.07736	2047.474		7158	7159	
114	173	287	Fl	2053.311319	2053.198		7154	7154	
114	174	288	Fl	2059.8252	2060.64		7152	7155	
114	175	289	Fl	2066.579426	2066.061		7150	7149	
115	173	288	Ef	2054.365367	2055.168		7133	7136	
115	174	289	Ef	2061.711572	2062.304		7134	7136	
115	175	290	Ef	2068.0048	2068.28		7131	7132	
115	176	291	Ef	2075.1223	2075.121		7131	7131	
116	173	289	Lv	2056.310388	2057.391		7115	7119	
116	174	290	Lv	2063.938	2064.8		7117	7120	
116	175	291	Lv	2070.481813	2070.756		7115	7116	
116	176	292	Lv	2077.7132	2078.164		7115	7117	
116	177	293	Lv	2084.089	2083.523		7113	7111	
117	175	292	Eh	2070.9419	2072.032		7092	7096	
117	176	293	Eh	2078.42155	2079.421		7094	7097	
117	177	294	Eh	2084.8576	2085.342		7091	7093	
118	175	293	Ei	2072.450485	2073.561		7073	7077	

References

1. Von Weizsäcker C F (1935) Zur Theorie der Kernmassen. Z Phys 96: 431- 458.
2. Bethe H A, R F Bacher (1936) Stationary states of nuclei. rev mod phys 8: 82.
3. Bohr A, Mottelson B R (1975) Nuclear structure 1.
4. Haustein P E (1988) An overview of the 1986–1987 atomic mass predictions. At Data Nucl Data Tables 39(2): 185-200.
5. P E Haustein in Particle Emission from Nuclei (CRC Press, Boca Raton, 1989) Vol (1) 233.
6. Myers W D, Swiatecki W J (1998) Nuclear diffuseness as a degree of freedom. Phys Rev C58: 3368.
7. Möller P, Nix J R, Myers W D, Swiatecki W J (1995) Nuclear ground-state masses and deformations. At Data Nucl Data Tables 59(2):185-381.
8. Möller P, Nix J R, Kratz K L (1997) Nuclear properties for astrophysical and radioactive-Ion-beam applications. at Data Nucl Data Tables 66(2):131-343.
9. Goriely S, Tondeur F, Pearson J M (2001) A Hartree–Fock nuclear mass table. at Data Nucl Data Tables 77(2): 311-381.
10. Ren Z (2002) Shape coexistence in even-even superheavy nuclei. Phys Rev C 65: 051304.
11. Ren Z, Tai F, Chen D H (2002) Systematic calculations of the ground state properties of superheavy nuclei. Phys Rev C 66: 064306.
12. Wang M, Audi G, Wapstra A H (2012) AME 2012 mass evaluation. CPC (HEP & NP) 36: 1603.
13. Bohr N, Wheeler J A (1939) The mechanism of nuclear fission. Phys Rev 56: 426.
14. Duflo J, Zuker A P (1995) Microscopic mass formulas. Phys Rev C 52(1): 23-27.
15. Liran S, Marinov A, Zeldes N (2000) Semiempirical shell model masses with magic number for superheavy elements. Phys Rev C 62: 047301.
16. Mittig W, Lepine-Szily A, Orr N A (1997) Mass measurement far from stability. Annu Rev Nucl Part Sci 47: 27-66.
17. Sarazin F, Savajols H, Mittig W (2000) Shape coexistence and the shell closure far from stability. Phys Rev Lett 84: 5062.
18. Honma M, Otsuka T, Brown B A, Mizusaki T (2002) Effective interaction for -shell nuclei. Phys Rev C 65: 061301.
19. Barea J, Frank A, Hirsch J G, Van Isacker P (2005) Nuclear masses set bounds on quantum chaos. Phys Rev Lett 94:102501.
20. Pahlavani M R, Sadeghi J, Morad R (2010) Binding energy of a holographic deuteron and tritium in anti-de-Sitter space/conformal field theory (AdS/CFT). Phys Rev C 82: 025201.
21. Pahlavani M R, Sadeghi J, Morad R (2011) Holographic 3He and 4He nuclei Jour of Phys G 38: 055002.
22. Pahlavani M R, Morad R (2013) Binding energy of light nuclei using the noncritical holography model. Phys Rev C 88: 064004.
23. Pahlavani M R, Morad R (2014) Application of AdS/CFT in nuclear physics. Advan in high ener phys 2014: 863268.
24. Xu C, Ren Z (2005) Systematical calculation of α decay half-life by density-dependent cluster modelNucl. Phys A 753(1-2): 174-185.
25. Swiatecki W J (1955) Systematics of spontaneous fission half-lives. phys rev 100: 937.
26. Sobiczewski A, Pomorski K (2007) Description of structure and properties of superheavy nucleiProg. in Part and Nucl Phys 58(1): 292-349.
27. Nilsson S G, Nix J R, Sobiczewski A, Szymański Z (1968) On the spontaneous fission of nuclei with Z near 114 and N near 184. Nucl Phys A 115(3): 545-562.
28. Myers W D, Swlatecki W J (1966) Nuclear masses and deformations nuclear physics 81(1): 1-60.
29. Dong T, Ren Z (2005) New model of binding energies of heavy nuclei with Z≥90. phys rev C 72: 064331.
30. MorettoL G (1972) Shell-model calculations of fission decay widths and probabilities in superheavy nuclei. Nucl Phys A 180(2): 337-362.
31. Cwiok S, Dobaczewski J, Heenen P H (1996) Shell structure of the superheavy elements. Nucl Phys A 611(1-2): 211-246.
32. Chasman R R, Ahmad I (1997) Nuclear structure at A~ 250 and the stability of the superheavy elements. phys lett B 392(3-4): 255-261.
33. Sobiczewski A, Muntian I, Patyk Z (2001) Calculated properties of superheavy nuclei. phys of atom nucl 64(6): 1105-1109.
34. Itkis M G, Oganessian Yu Ts, Zagrebaev V I (2002) Fission barriers of superheavy nuclei. Phys Rev C 65: 044602.